Frontiers of Hadronic Physics:
Brain circulation kick off workshop
March 21, 2013, BNL

Heavy quarkonium potential with almost physical quark masses from lattice QCD

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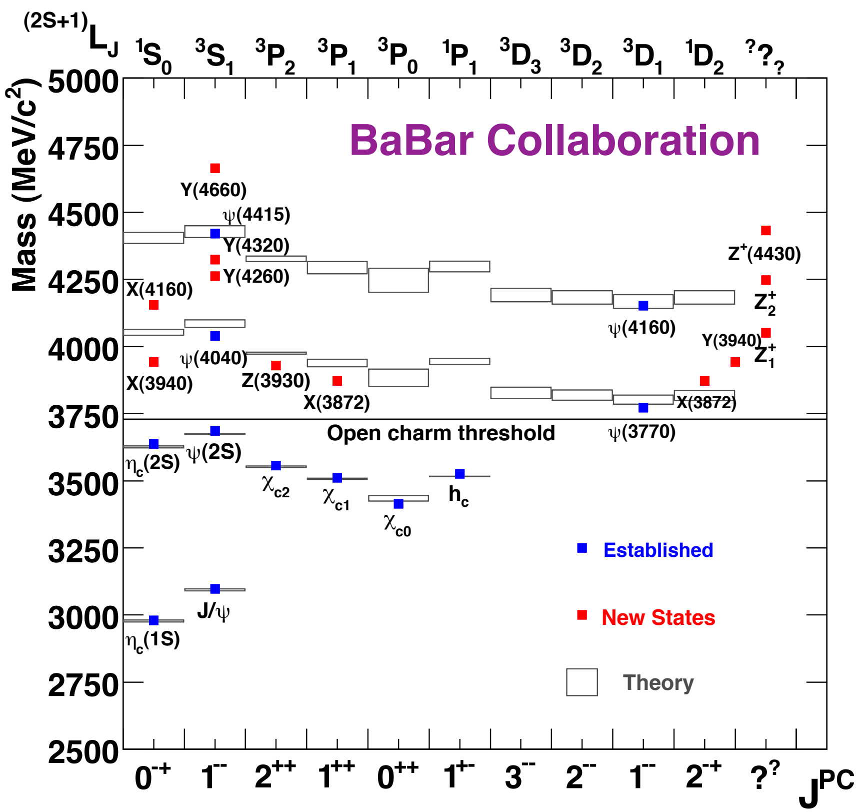
in collaboration with Shoichi Sasaki (Tohoku University)

T. Kawanai and S. Sasaki, PRL, 107, 091601 (2011)
T. Kawanai and S. Sasaki, PRD85, 091503(R) (2012)

Why cc^{bar} potential？

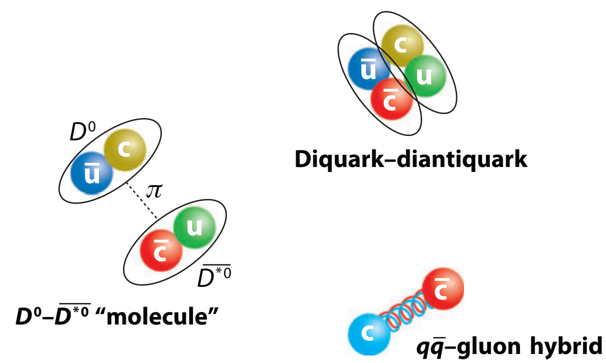
✦ Exotic XYZ charmonium-like mesons

“Standard” states can be defined in potential models



The XYZ mesons are expected to be good candidates for non-standard quarkonium mesons

S. Godfrey and S. L. Olsen,
Ann. Rev. Nucl. Part. Sci. 58, 51 (2008)



“Exotic” = “Non-standard”?

Why $c\bar{c}$ potential？

✦ $q\bar{q}$ interquark potential in quark models

S. Godfrey and N. Isgur, PRD 32, 189 (1985).
T. Barnes, S. Godfrey and E. S. Swanson, PRD 72, 054026 (2005)

$$V_{c\bar{c}} = \underbrace{-\frac{4}{3}\frac{\alpha_s}{r} + \sigma r}_{\text{Cornell potential}} + \underbrace{\frac{32\pi\alpha_s}{9m_q^2}\delta(r)\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}} + \frac{1}{m_q^2}\left[\left(\frac{2\alpha_s}{r^3} - \frac{b}{2r}\right)\mathbf{L}\cdot\mathbf{S} + \frac{4\alpha_s}{r^3}T\right]}_{\text{spin-dependent potential}}$$

- Spin-spin, tensor and spin-orbit terms appear as corrections in the $1/m_q$ expansion.
- Functional forms of the spin-dependent terms are determined by one-gluon exchange.

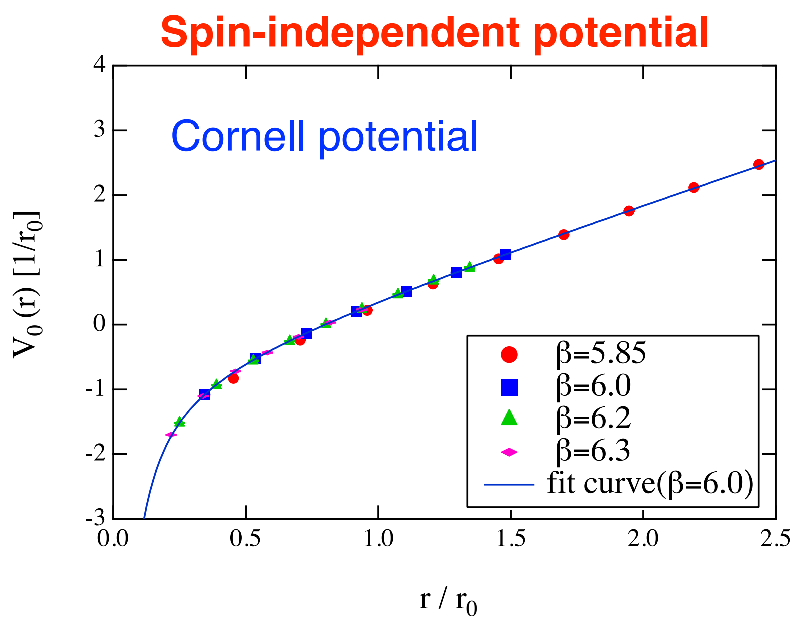
→ Properties of higher charmonium states predicated
in potential models may suffer from large uncertainties.

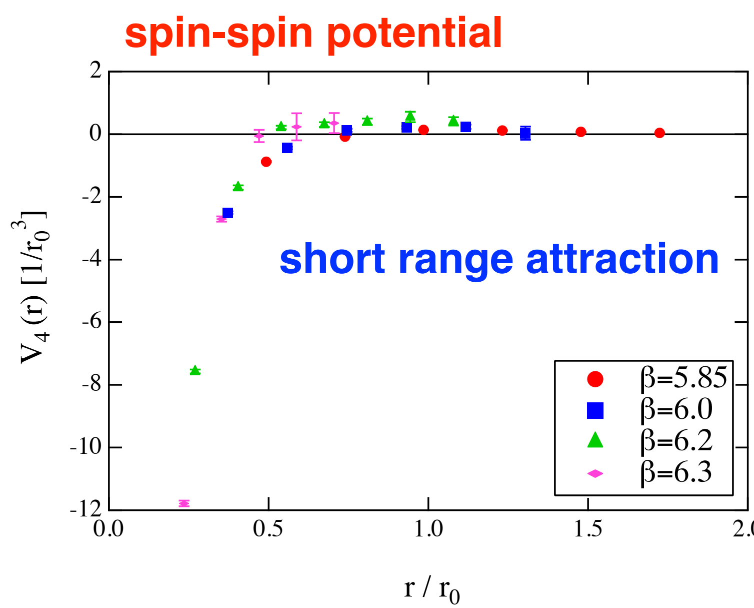
A reliable charmonium potential directly derived
from first principles QCD is very important.

Why $c\bar{c}$ potential？

✦ Static interquark potential from Wilson loop

G. S. Bali, Phys. Rept. 343, 1 (2001).





- The static potential have been precisely calculated by Wilson loop from lattice QCD.
- Relativistic corrections are classified in powers of $1/m_q$ within framework of pNRQCD.

N. Brambilla et al., Rev. Mod. Phys. 77, 1423 (2005).

→ spin-spin potential induced by $1/m_q^2$ correction exhibits short range attraction.
cf. short range repulsion is required in phenomenology.

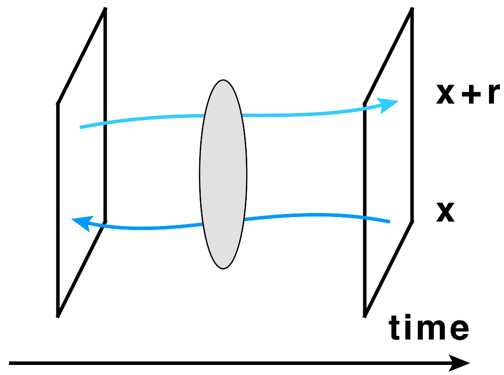
Koma et al., NPB769 (2007) 79
Koma et al., PRL97 (2006) 122003

How to calculate cc^{bar} potential？

S. Aoki, T. Hatsuda and N. Ishii, Prog. Theor. Phys. 123, 89(2010).
Y. Ikeda and H. Iida, Prog.Theor.Phys.Suppl. 186, 228 (2010)

1. Equal-time BS wavefunction

$$\phi_\Gamma(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0|\bar{q}(\mathbf{x})\Gamma q(\mathbf{x}+\mathbf{r})|q\bar{q}; J^{PC}\rangle$$



$$\sum_{\mathbf{x},\mathbf{x}',\mathbf{y}'} \langle 0|\bar{q}(\mathbf{x},t)\Gamma q(\mathbf{x}+\mathbf{r},t)\left(\bar{q}(\mathbf{x}',t_{\rm src})\Gamma q(\mathbf{y}',t_{\rm src})\right)^\dagger|0\rangle$$
$$= \sum_n A_n \langle 0|\bar{q}(\mathbf{x})\Gamma q(\mathbf{x}+\mathbf{r})|n\rangle e^{-M_n^\Gamma(t-t_{\rm src})}$$
$$\xrightarrow{t\gg t_0} A_0 \phi_\Gamma(\mathbf{r}) e^{-M_0^\Gamma(t-t_{\rm src})}$$

2. Schrödinger equation with non-local potential

$$-\frac{\nabla^2}{2\mu}\phi_\Gamma(\mathbf{r}) + \int dr' U(\mathbf{r},\mathbf{r}')\phi_\Gamma(\mathbf{r}') = E_\Gamma \phi_\Gamma(\mathbf{r})$$

3. Velocity expansion

$$U(\mathbf{r}',\mathbf{r}) = \{V(r) + V_{\rm S}(r)\mathbf{S}_Q\cdot\mathbf{S}_{\overline{Q}} + V_{\rm T}(r)S_{12} + V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S} + \mathcal{O}(\nabla^2)\}\delta(\mathbf{r}'-\mathbf{r})$$

How to calculate cc^{bar} potential？

S. Aoki, T. Hatsuda and N. Ishii, Prog. Theor. Phys. 123 (2010) 89.
Y. Ikeda and H. Iida, arXiv:1102.2097 [hep-lat].

5. Projection to “S-wave” $\phi_\Gamma(\mathbf{r}) \rightarrow \phi_\Gamma(r; A_1^+)$

$$\left\{-\frac{\nabla^2}{m_q} + V(r) + \mathbf{S}_q \cdot \mathbf{S}_{\overline{q}} V_{\mathrm{S}}(r)\right\} \phi_\Gamma(r) = E_\Gamma \phi_\Gamma(r)$$

6. Linear combination

$$\begin{aligned} V(r) &= E_{\mathrm{ave}} + \frac{1}{m_q}\left\{\frac{1}{4}\frac{\nabla^2\phi_{\mathrm{PS}}(r)}{\phi_{\mathrm{PS}}(r)} + \frac{3}{4}\frac{\nabla^2\phi_{\mathrm{V}}(r)}{\phi_{\mathrm{V}}(r)}\right\} \\ V_{\mathrm{S}}(r) &= E_{\mathrm{hyp}} + \frac{1}{m_q}\left\{-\frac{\nabla^2\phi_{\mathrm{PS}}(r)}{\phi_{\mathrm{PS}}(r)} + \frac{\nabla^2\phi_{\mathrm{V}}(r)}{\phi_{\mathrm{V}}(r)}\right\} \end{aligned}$$

The quark kinetic mass m_q is essentially involved in the definition of the potentials.
Under a simple, but reasonable assumption of $\lim_{r\to\infty} V_S(r) = 0$

T. Kawanai and S. Sasaki, PRL. 107, 091601 (2011).

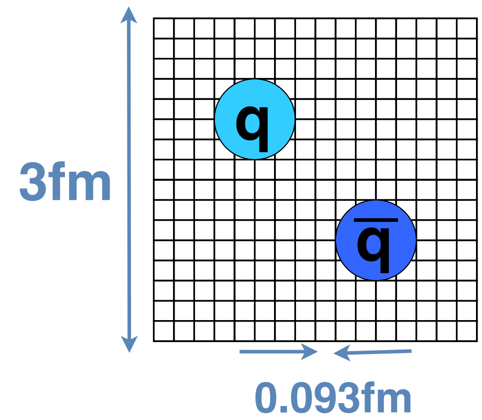
$$m_q = \lim_{r\to\infty} \frac{-1}{\Delta E_{\mathrm{hyp}}}\left(\frac{\nabla^2\phi_{\mathrm{V}}(r)}{\phi_{\mathrm{V}}(r)} - \frac{\nabla^2\phi_{\mathrm{PS}}(r)}{\phi_{\mathrm{PS}}(r)}\right)$$

$$\Delta E_{\mathrm{hyp}} = M_{\mathrm{V}} - M_{\mathrm{PS}}$$

1. Quenched lattice QCD simulation

2. N_f =2+1 dynamical QCD simulation

Lattice Set up

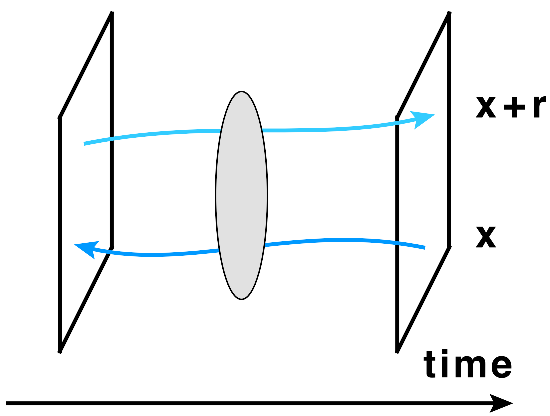


- Quenched QCD (No dynamical quarks)
- Lattice size : $L^3 \times T = 32^3 \times 48$ (~3fm^3)
- plaquette gauge action β=6.0 (a=0.093 fm, a^{-1}=2.1GeV)

  \+ RHQ action with tad-pole improved one-loop PT coefficients

  Y. Kayaba et al. [CP-PACS Collaboration], JHEP 0702, 019 (2007).
- 6 hopping parameters : $0.06667 \leq \kappa_Q \leq 0.11456$

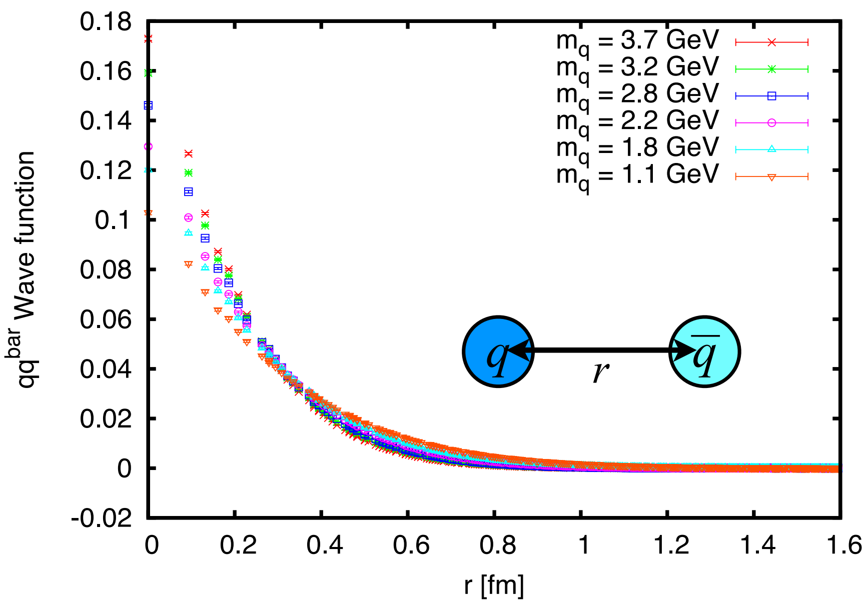
  $1.87\ \text{GeV} \leq m_{pseudo} \leq 5.83\ \text{GeV}$
- Statistics : 150 configs
- Wall source
- Coulomb gauge fixing



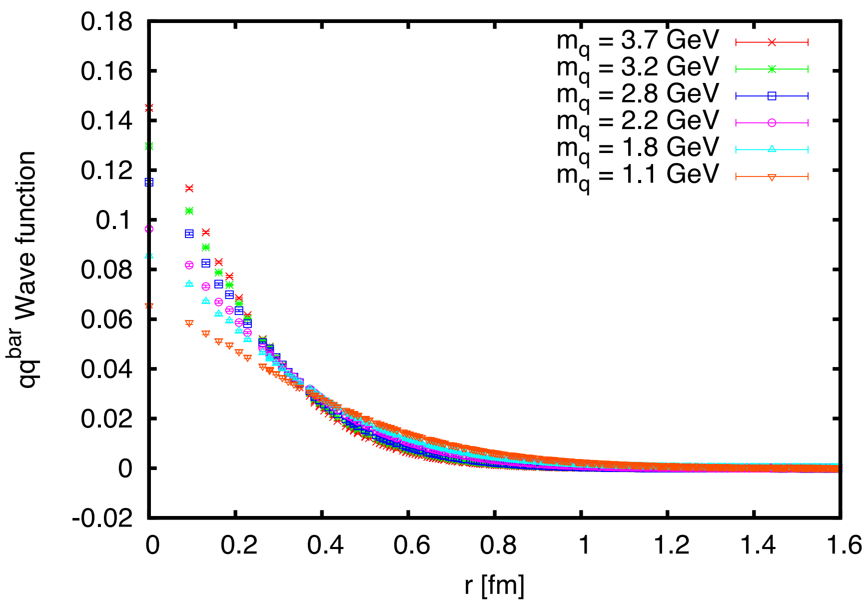
$q\bar{q}$ wave function

$$\phi_\Gamma(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0|\bar{q}(\mathbf{x})\Gamma q(\mathbf{x}+\mathbf{r})|q\bar{q}; J^{PC}\rangle$$

Pseudo scalar $J^P = 0^-$



Vector $J^P = 1^-$

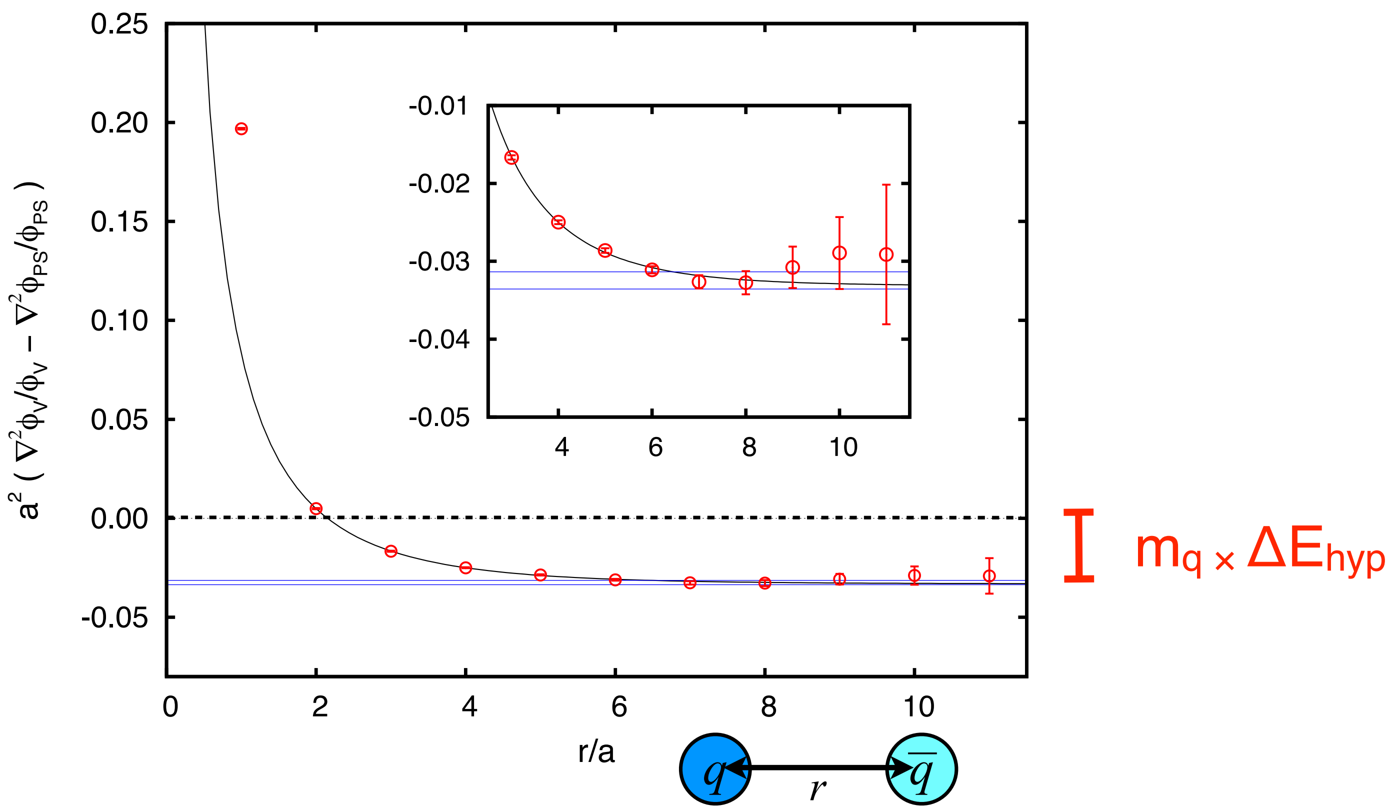


- Normalization $\int dr^3\, \psi^2(r) = 1$
- BS wave functions vanish at $r \sim 1$fm
- Size of wave function with heavier quark mass become smaller.

Determination of kinetic quark mass

T. Kawanai and S. Sasaki, PRL. 107, 091601 (2011)

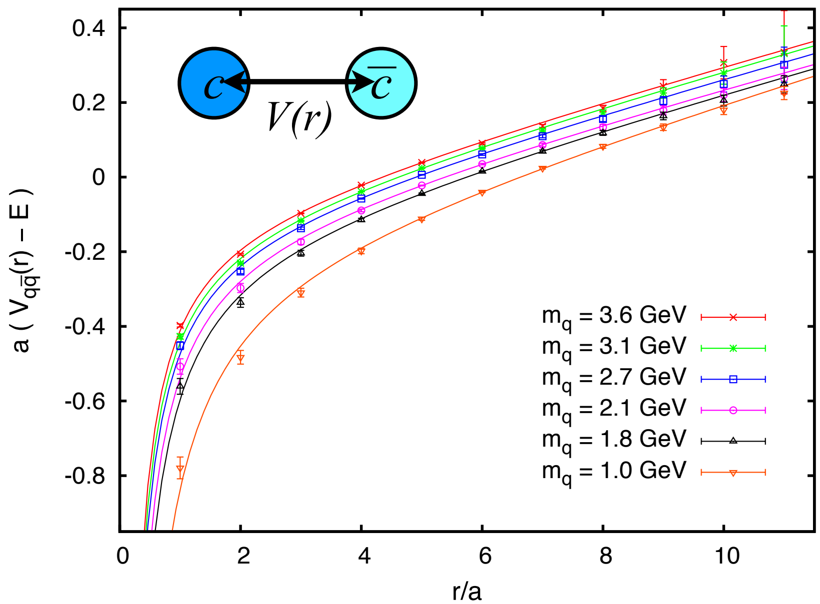
$$m_q = \lim_{r\to\infty} \frac{-1}{\Delta E_{\rm hyp}} \left(\frac{\nabla^2 \phi_{\rm V}(r)}{\phi_{\rm V}(r)} - \frac{\nabla^2 \phi_{\rm PS}(r)}{\phi_{\rm PS}(r)} \right)$$



spin-independent qq^{bar} potential

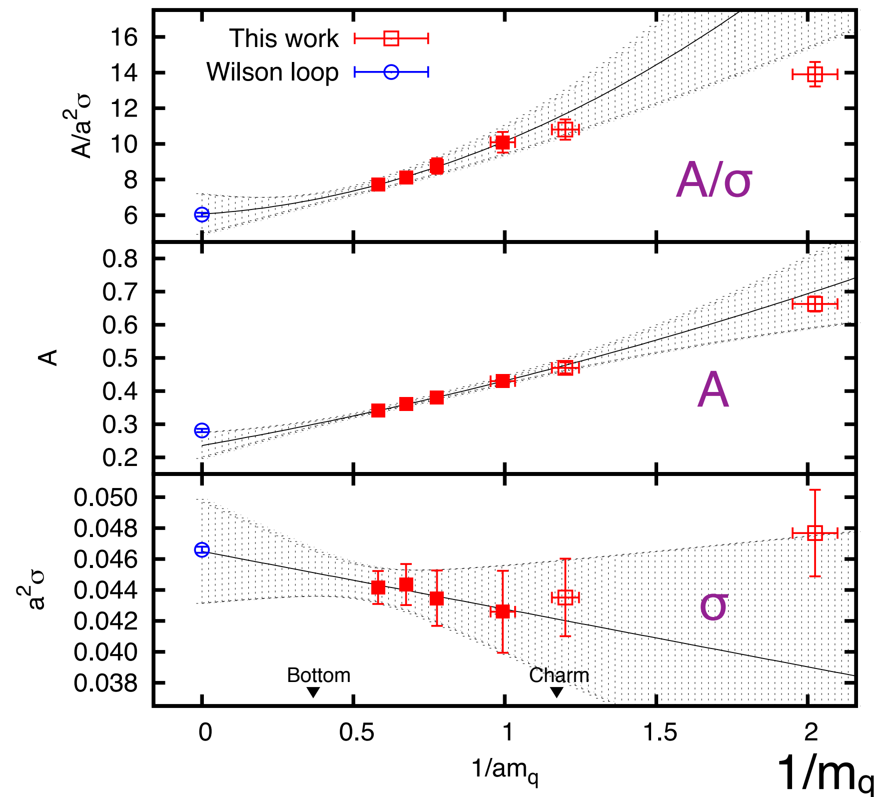
T. Kawanai and S. Sasaki, PRL. 107, 091601 (2011)

- Quark mass dependence



Cornell parameterization

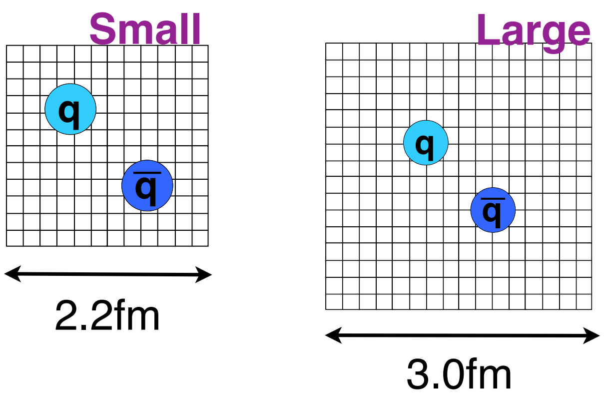
$$V(r) = -\frac{A}{r} + \sigma r + V_0$$

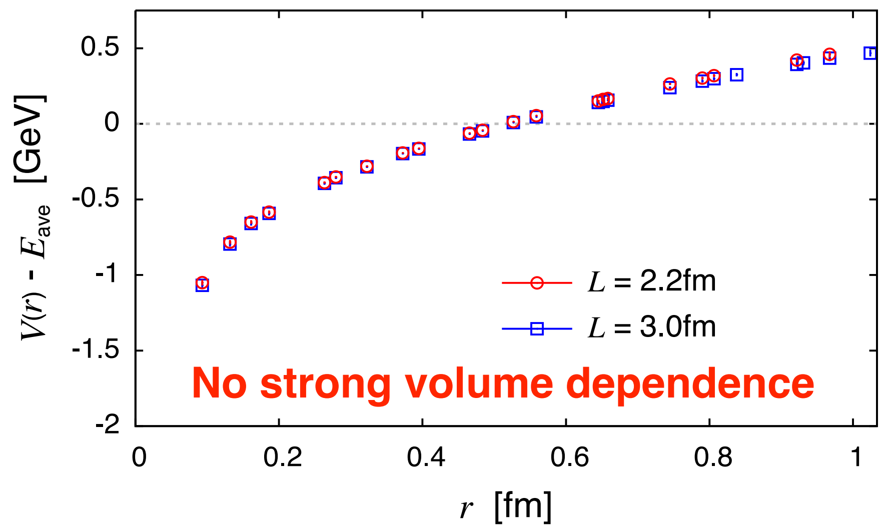


Consistent with the Wilson loops in the $m_q \to \infty$ limit

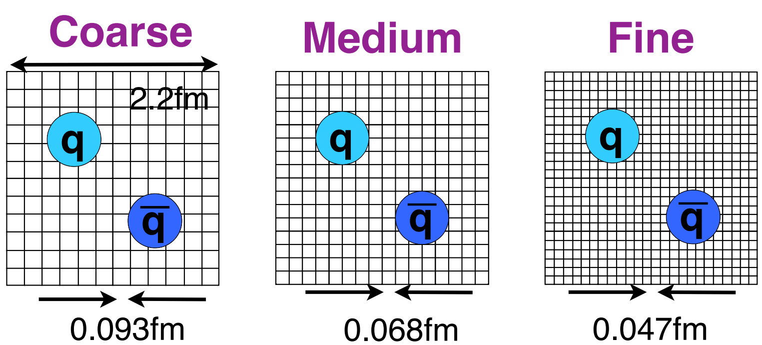
spin-independent qq$^{\text{bar}}$ potential

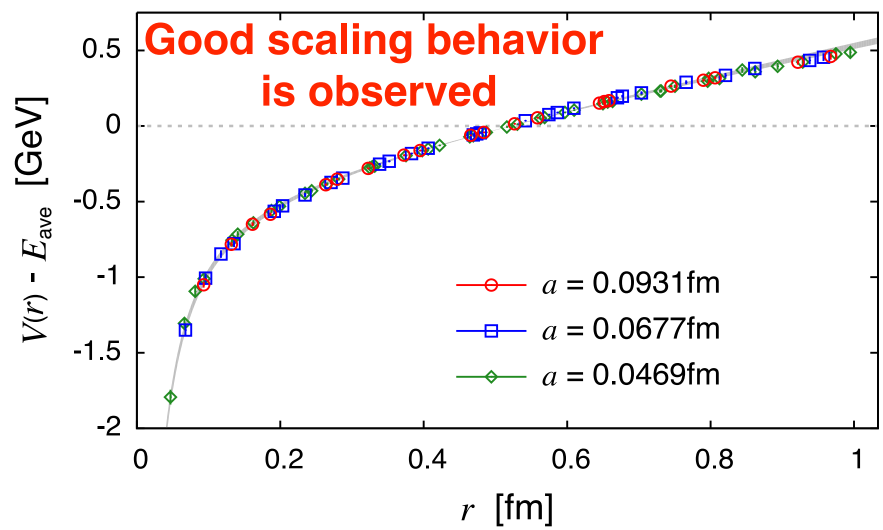
- Volume dependence





- Finite lattice spacing effects

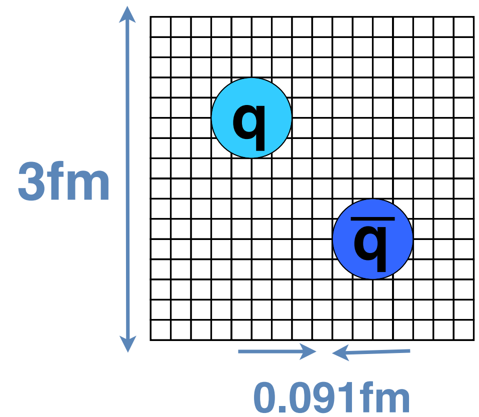




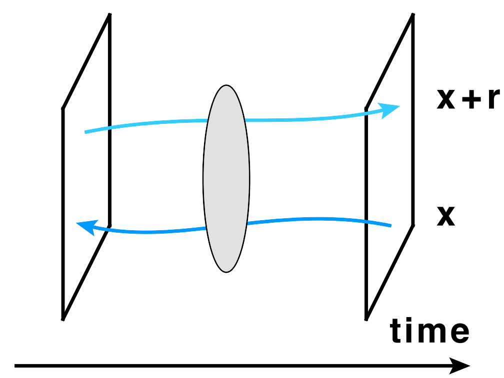
1. Quenched lattice QCD simulation

2. N_f =2+1 dynamical QCD simulation

Lattice Set up

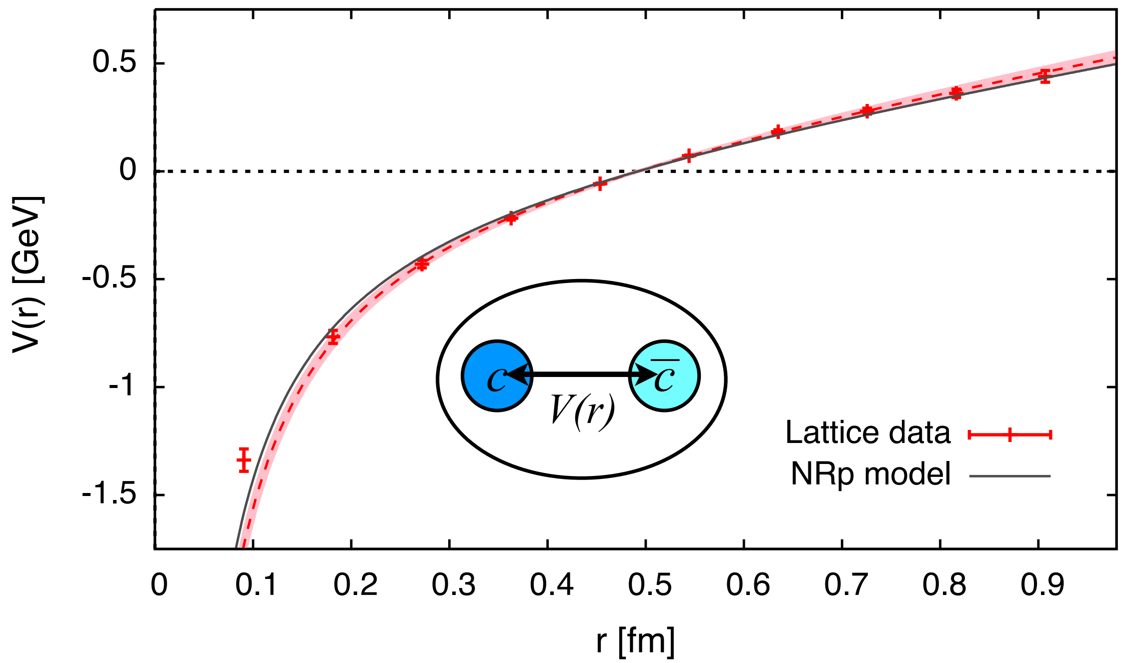


- 2+1 flavor dynamical gauge configurations generated by PACS-CS collaboration.
- Lattice size : $L^3 \times T = 32^3 \times 64$ (~3fm^3)
- Iwasaki gauge action $\beta=1.9$ ($a \approx 0.091$ fm, $a^{-1} \approx 2.3$GeV) + RHQ action with partially non-perturbative RHQ parameters.
- Light quark mass : m_π = 156(7) MeV, m_K = 553(2)MeV
  Charm quark mass : $m_{ave}(1S)$ =3.069(2) GeV, $m_{hyp}(1S)$=111(2) MeV
- Statistics : 198 configs
- Wall source
- Coulomb gauge fixing



spin-independent cc^{bar} potential

T. Kawanai and S. Sasaki, arXiv:1110.0888



	This work	NRp model	Static
A	0.813(22)	0.7281	0.403(24)
$\sqrt{\sigma}$ [GeV]	0.394(7)	0.3775	0.462(4)
m_q [GeV]	1.74(7)	1.4794	∞

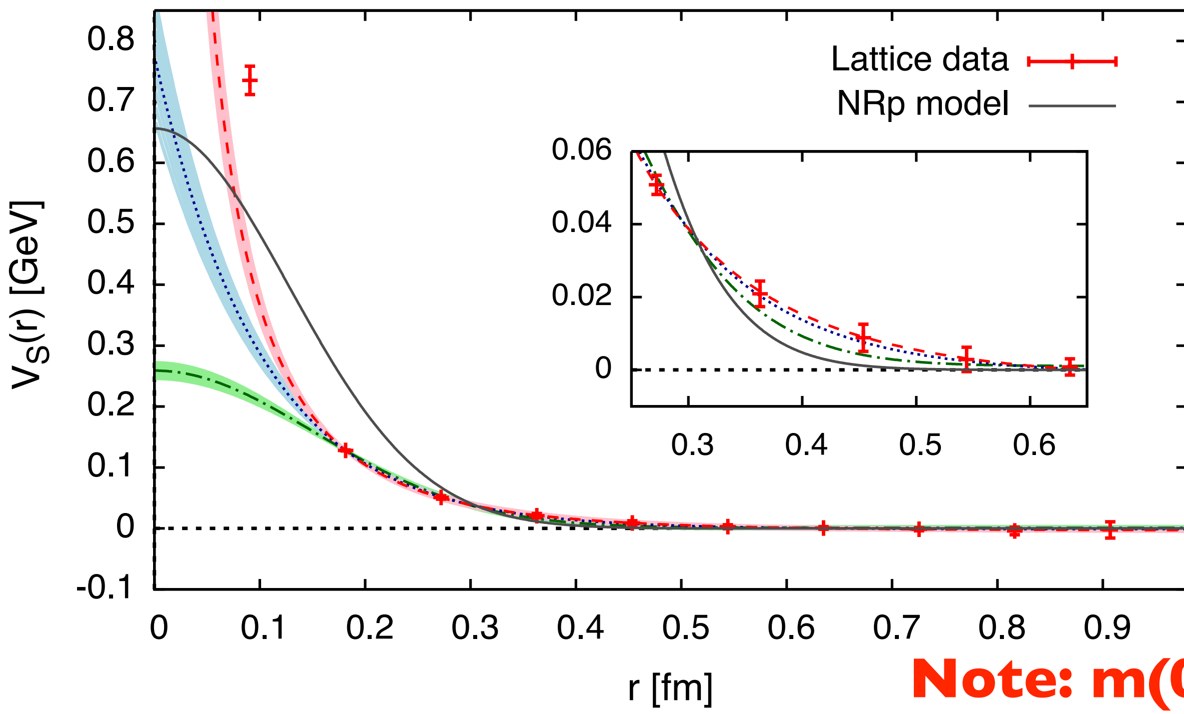
▶ The charmonium potential obtained from the BS wave function resembles one used in the NRp model.

▶ String breaking is not observed

Non-relativistic potential (NRp) model
T.Barnes, S. Godfrey, E.S. Swanson, PRD72 (2005) 054026

spin-spin $c\bar{c}$ potential

T. Kawanai and S. Sasaki, arXiv:1110.0888



Note: m(0$^-$) < m(1$^-$)

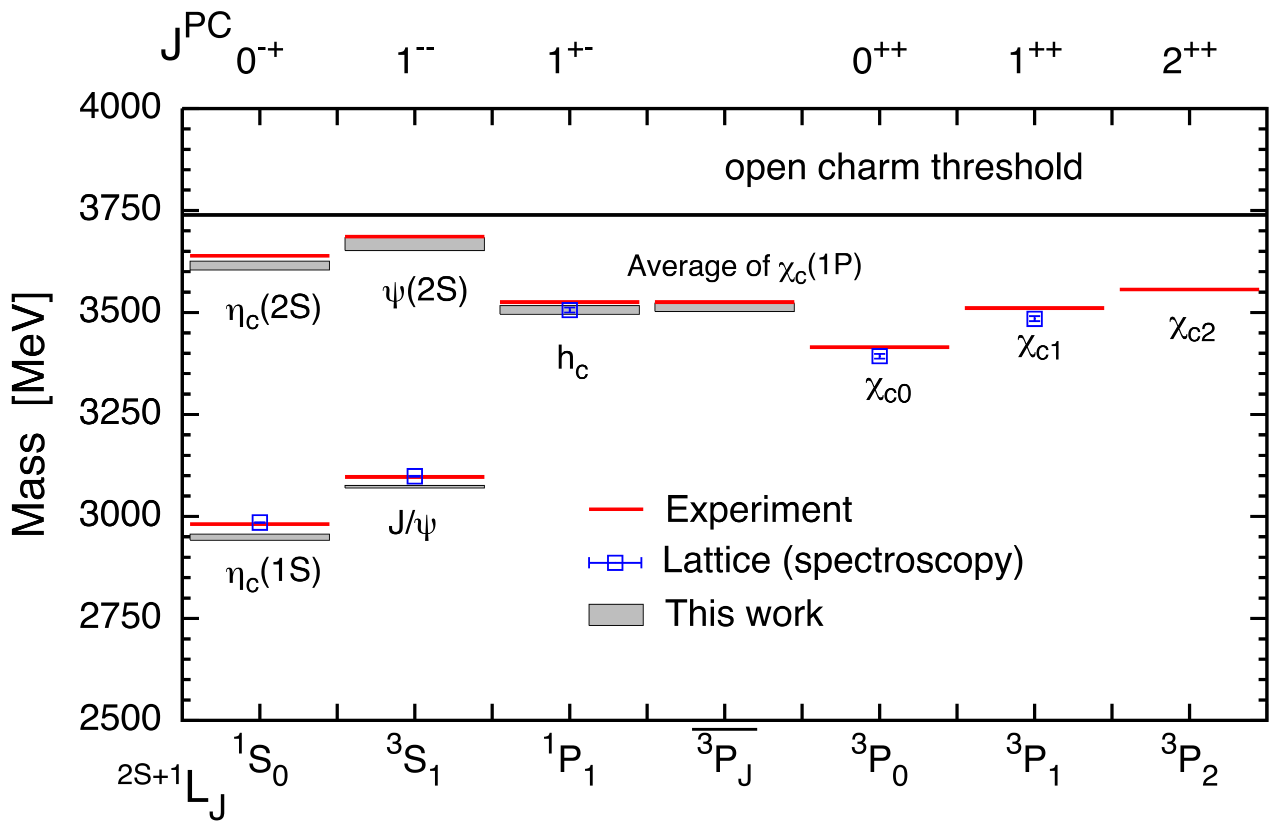
- Short range, but non-point like, repulsive interaction
- A difference appears in the spin-spin potential

Fitting function

$$V_S(r) = \begin{cases} \alpha \exp(-\beta r)/r \\ \alpha \exp(-\beta r) \\ \alpha \exp(-\beta r^2) \end{cases}$$

	α	β	χ/ d.o.f
Yukawa	0.297(12)	0.982(47) GeV	0.89
exponential	0.866(29) GeV	2.067(37) GeV	0.45
Gaussian	0.309(7) GeV	1.069(17) GeV^2	12.40

charmonium mass spectrum below open-charm threshold

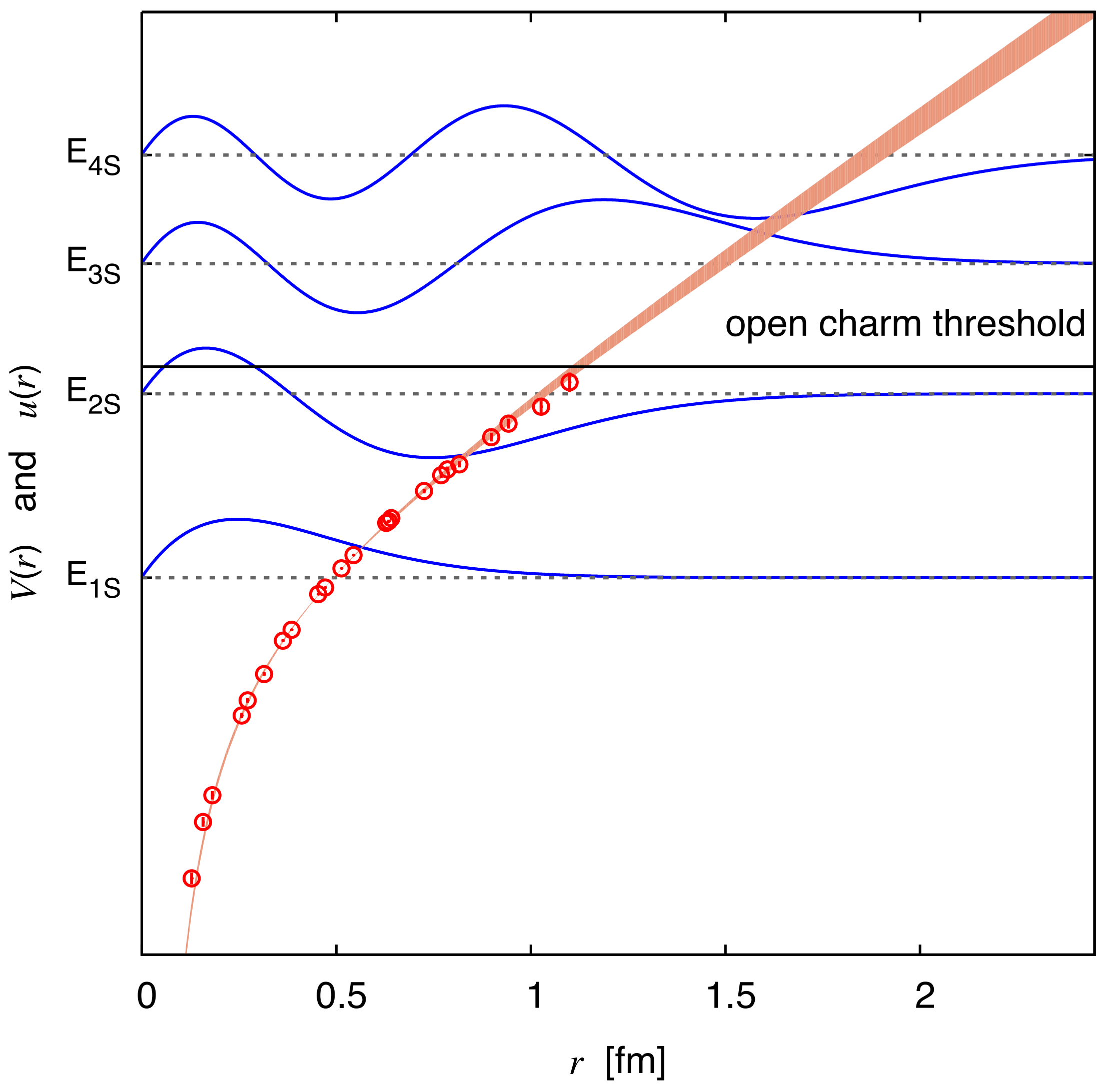


Low-lying charmonium masses obtained from the quark potential model with lattice inputs are in good agreement with the experimental measurements.

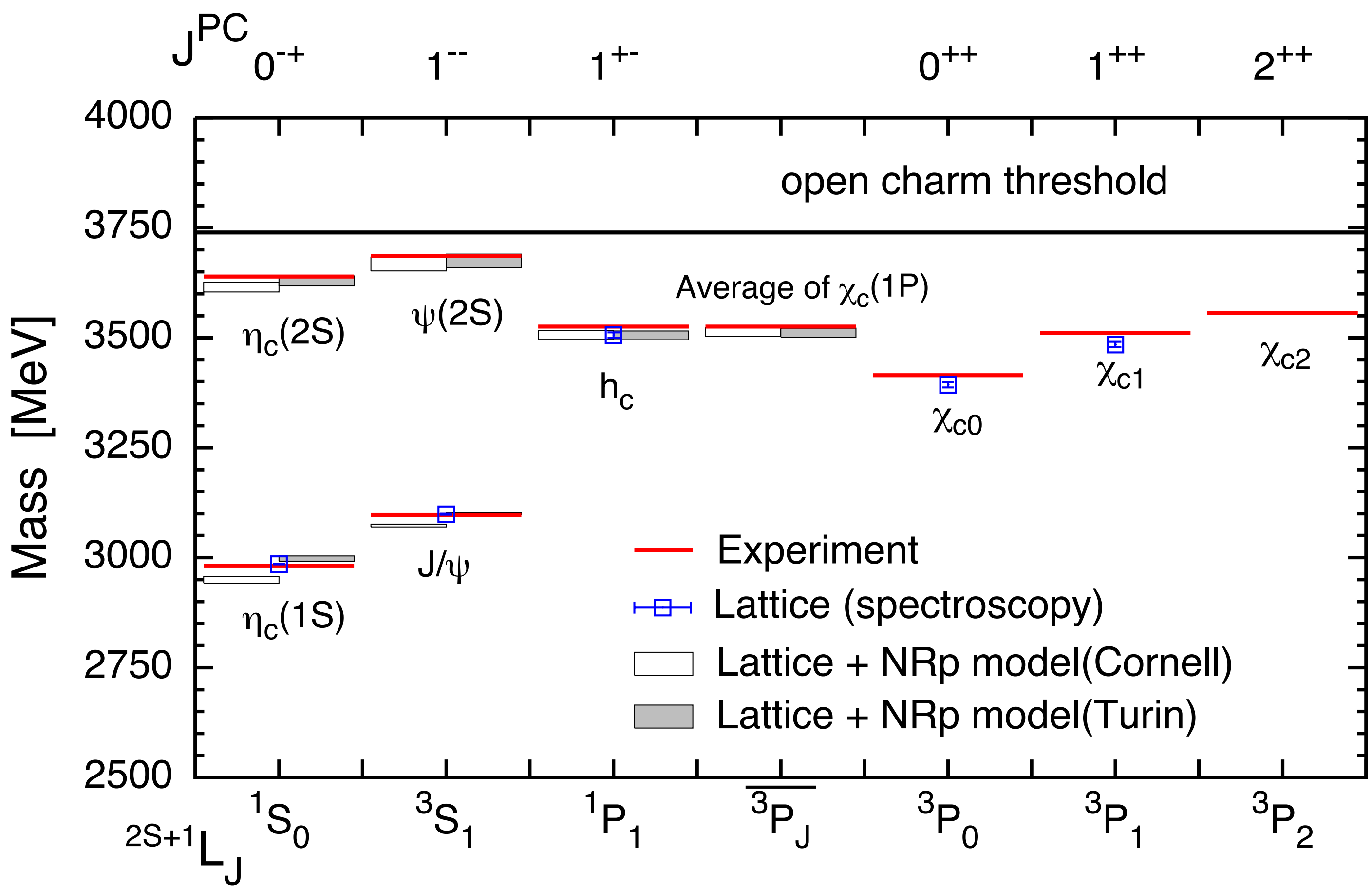
Summary

- ✦ We have derived both the spin-independent and -dependent part of the central $q\bar{q}$ interquark potential from the BS wave function in Quenched QCD simulation and 2+1 flavor dynamical lattice QCD simulation with almost physical quark masses.
  - ✓ spin-independent $q\bar{q}$ potential from BS wave function smoothly approaches the static $q\bar{q}$ potential from Wilson loop.
  - ✓ The spin-independent charmonium potential obtained from the BS wave function resembles the one used in the NRp model.
  - ✓ Spin-spin potential from lattice QCD shows the repulsive interaction.
  - ✓ The resulting charmonium potential can reproduce experimental spectroscopy.
- ✦ Future perspective
  - ✓ Other spin-dependent potential: tensor and LS force.
  - ✓ To extend the bottomonium (Now under way)

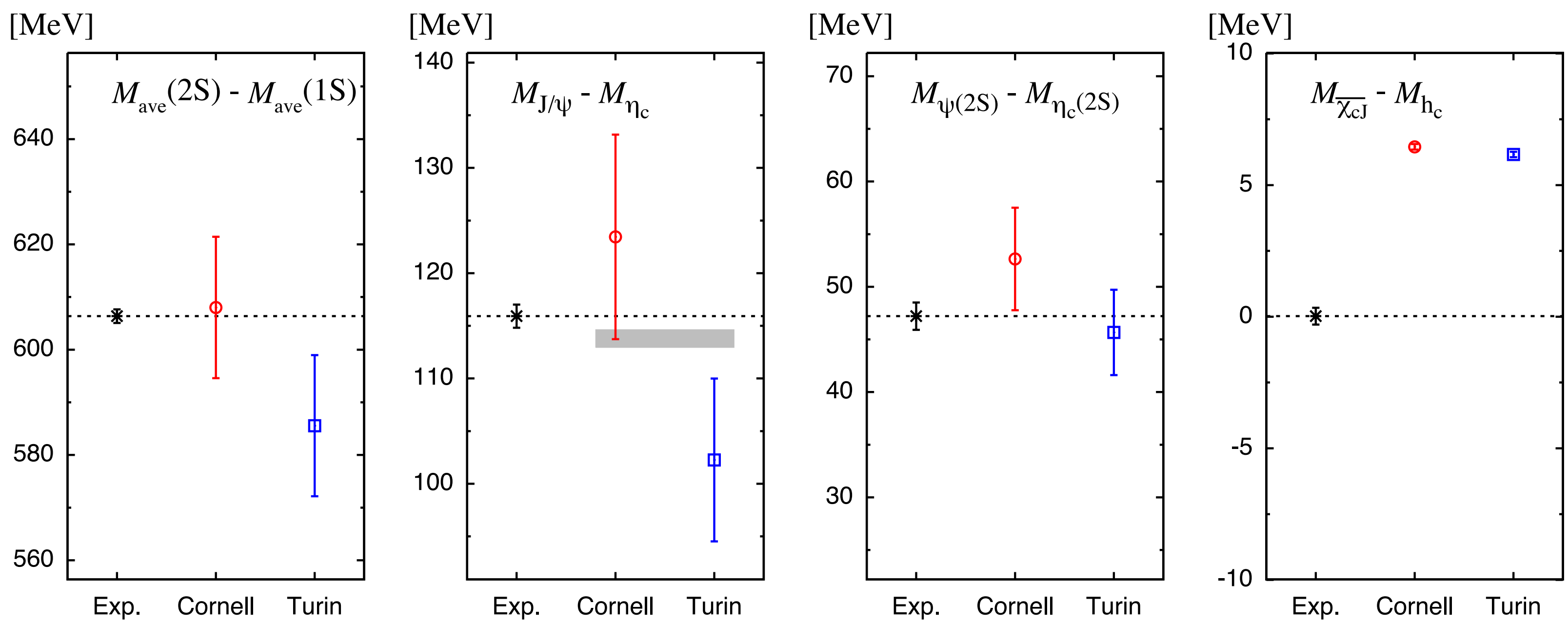
Thank you for your attention.

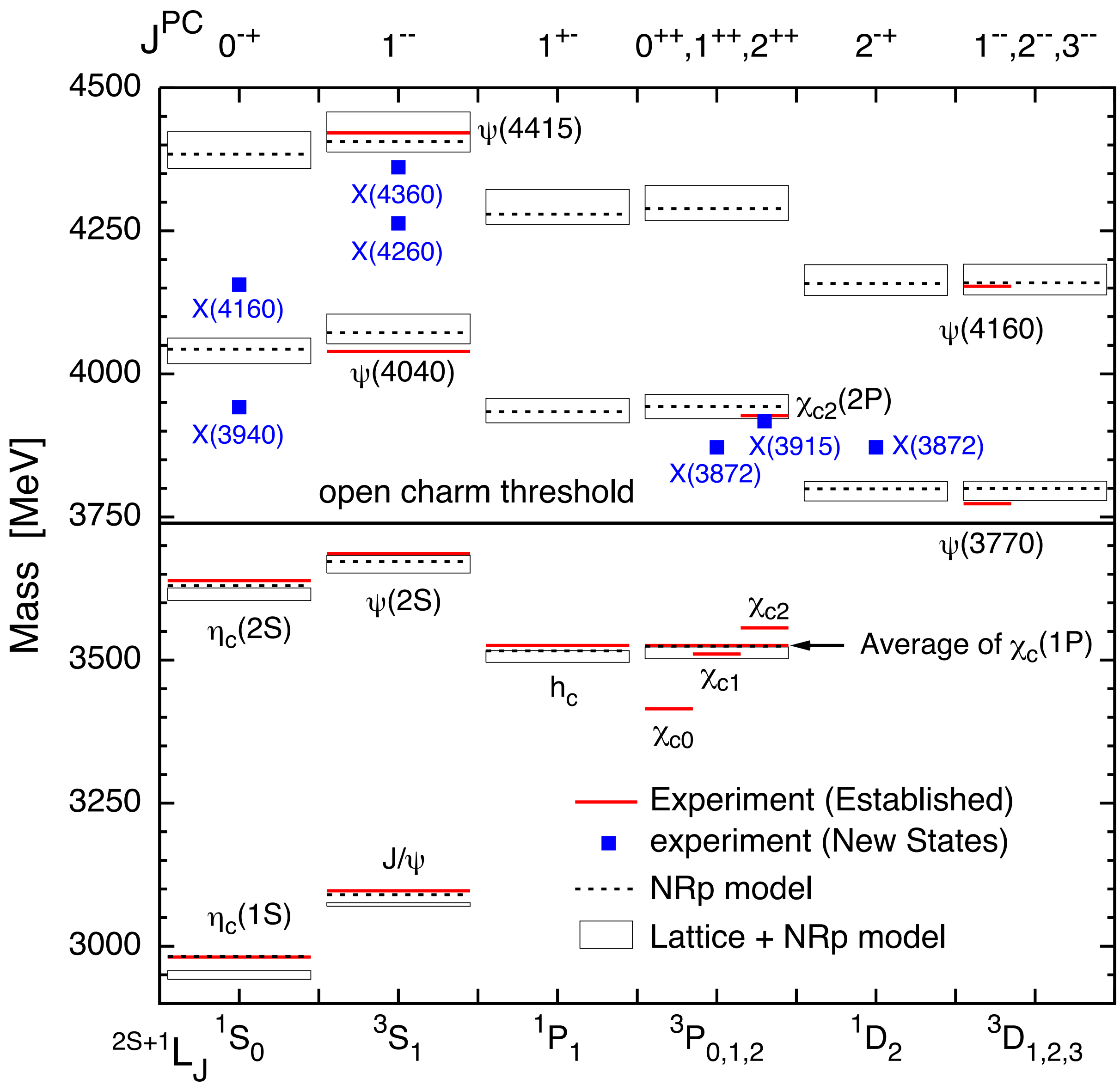


E4S
E3S
E2S
E1S
open charm threshold
V(r) and u(r)
0
0.5
1
1.5
2
r [fm]



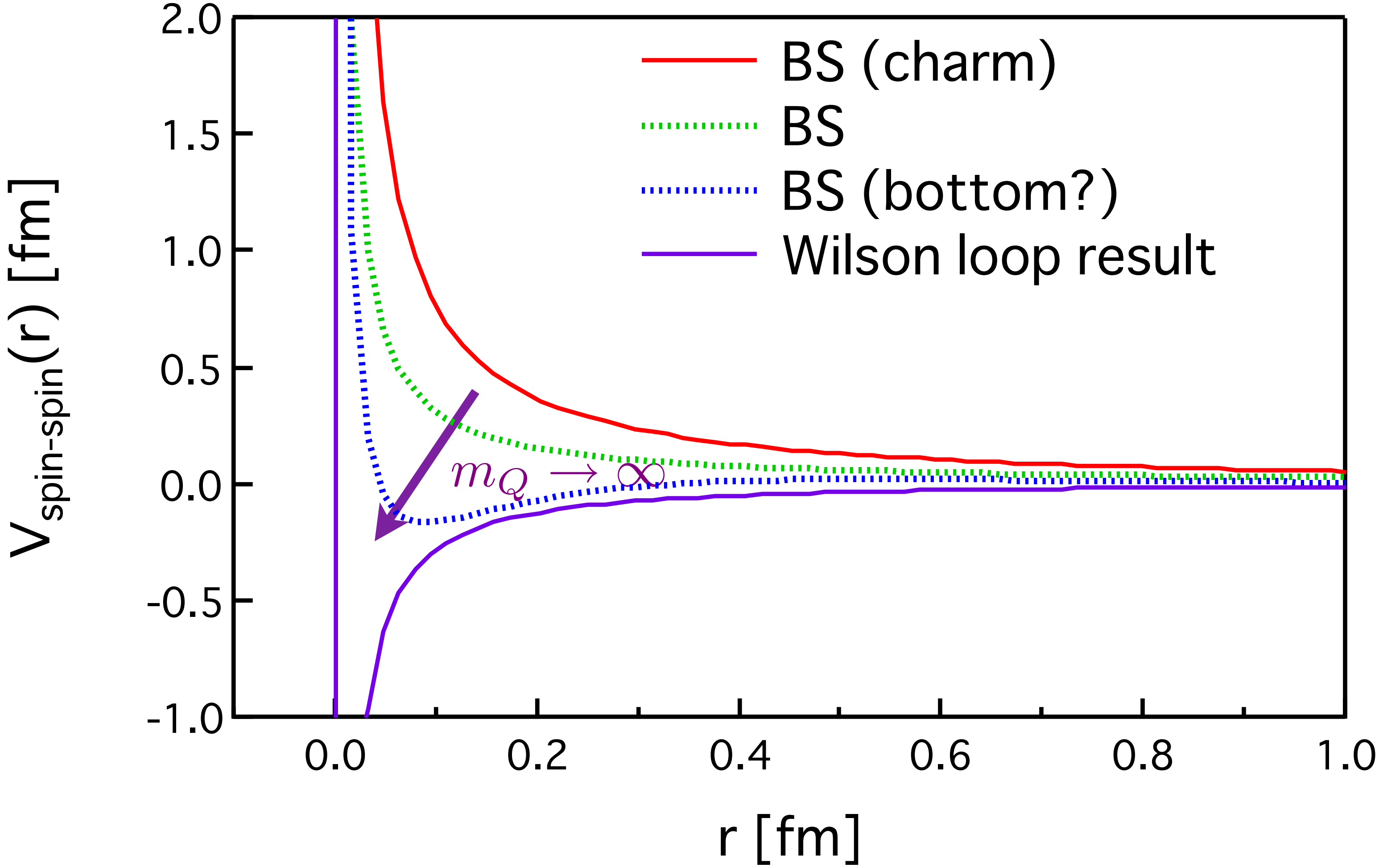
J^{PC}
0^{-+}
1^{--}
1^{+-}
0^{++}
1^{++}
2^{++}
4000
3750
3500
3250
3000
2750
2500
Mass [MeV]
open charm threshold
Average of $\chi_c(1P)$
$\eta_c(2S)$
$\psi(2S)$
h_c
χ_{c0}
χ_{c1}
χ_{c2}
$\eta_c(1S)$
J/ψ
Experiment
Lattice (spectroscopy)
Lattice + NRp model(Cornell)
Lattice + NRp model(Turin)
$^{2S+1}L_J$
$^1S_0$
$^3S_1$
$^1P_1$
$\overline{^3P_J}$
$^3P_0$
$^3P_1$
$^3P_2$


[MeV]
$M_{ave}(2S)$ - $M_{ave}(1S)$
640
620
600
580
560
Exp.
Cornell
Turin
[MeV]
$M_{J/\psi}$ - M_{η_c}
140
130
120
110
100
Exp.
Cornell
Turin
[MeV]
$M_{\psi(2S)}$ - $M_{\eta_c(2S)}$
70
60
50
40
30
Exp.
Cornell
Turin
[MeV]
$M_{\overline{\chi_{cJ}}}$ - M_{h_c}
10
5
0
-5
-10
Exp.
Cornell
Turin



J^PC
0^-+
1^--
1^+-
0^++,1^++,2^++
2^-+
1^--,2^--,3^--
4500
4250
4000
3750
3500
3250
3000
Mass [MeV]
ψ(4415)
X(4360)
X(4260)
X(4160)
ψ(4160)
ψ(4040)
χc2(2P)
X(3940)
X(3915)
X(3872)
X(3872)
open charm threshold
ψ(3770)
ψ(2S)
ηc(2S)
χc2
Average of χc(1P)
χc1
hc
χc0
Experiment (Established)
experiment (New States)
NRp model
Lattice + NRp model
J/ψ
ηc(1S)
2S+1LJ
1S0
3S1
1P1
3P0,1,2
1D2
3D1,2,3

Conjecture

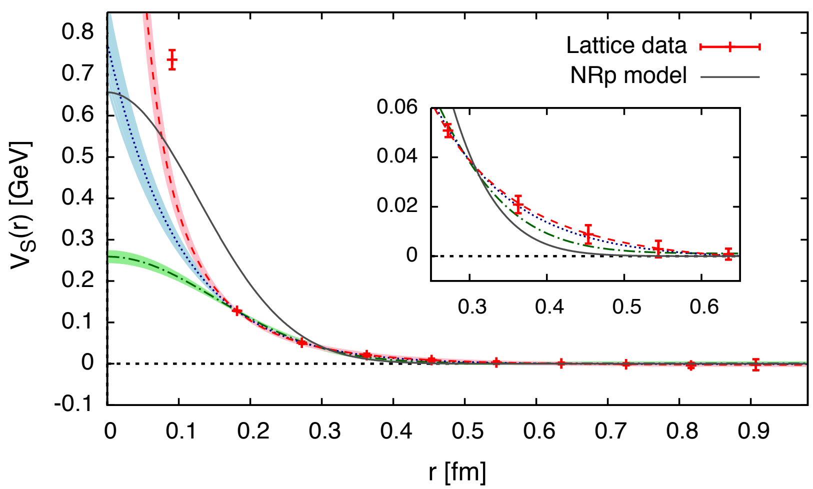


BS (charm)
BS
BS (bottom?)
Wilson loop result
$m_Q \to \infty$
$V_{\text{spin-spin}}(r)$ [fm]
r [fm]
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
0.0
0.2
0.4
0.6
0.8
1.0

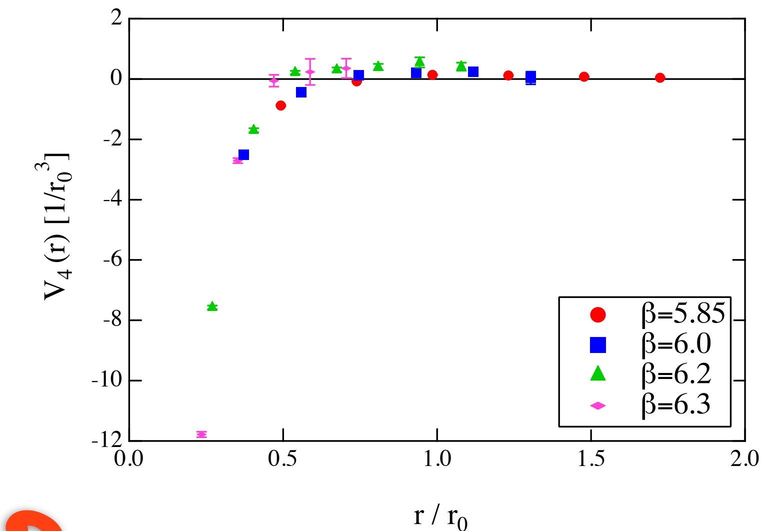
Comment on spin-spin potential

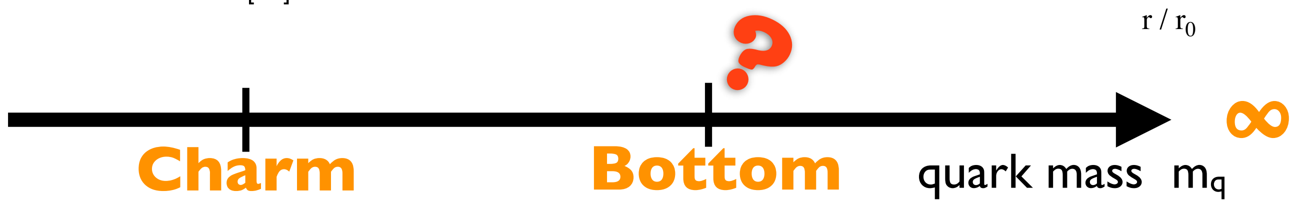
Spin-spin potentials calculated from lattice QCD show the quite different qualitative behavior between our approach and wilson loop approach.

Our approach



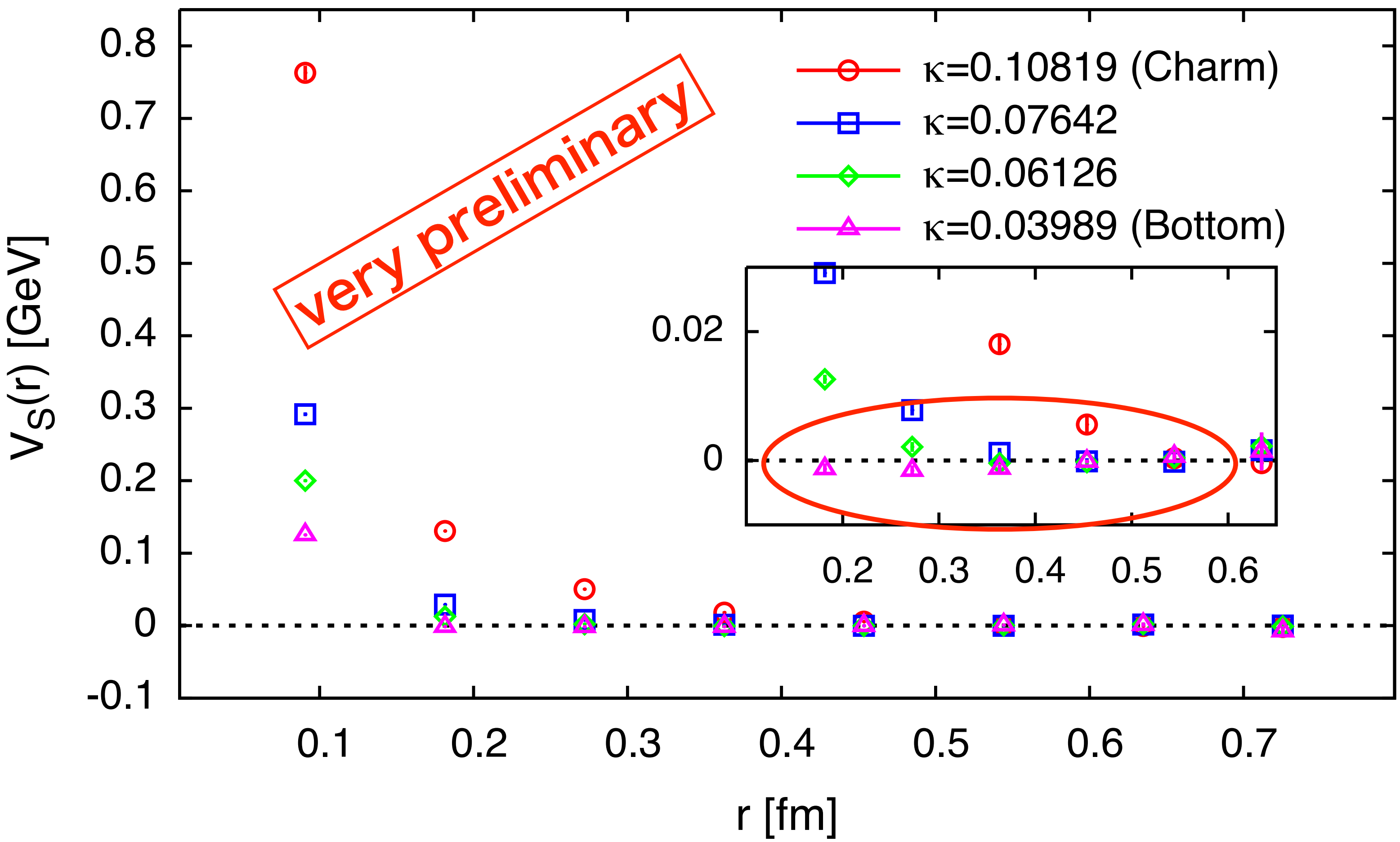
Wilson loop approach





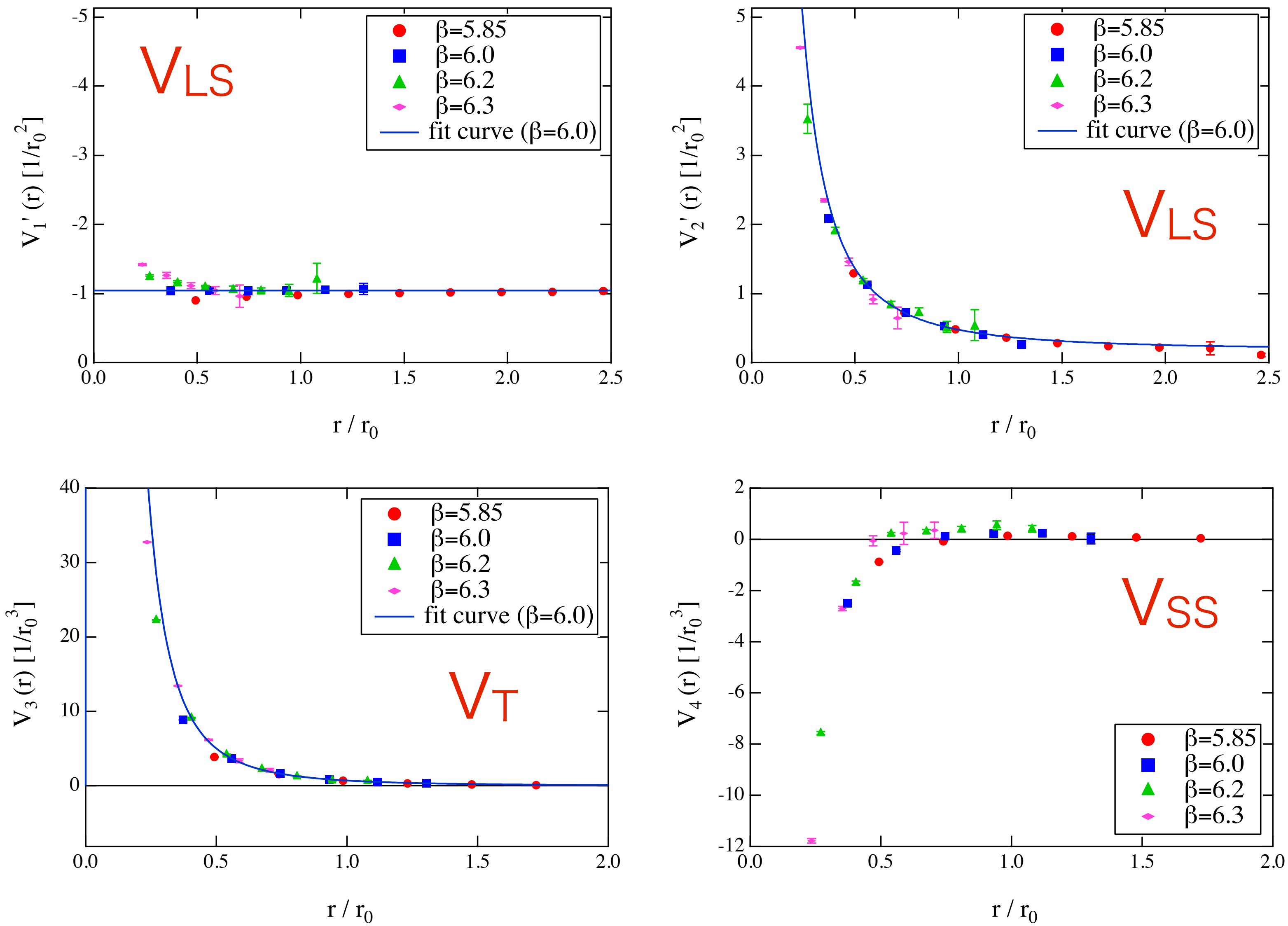
Dose finite quark mass effect change the spin-spin potential from attractive to repulsive ?

spin-spin bb^{bar} potential



Spin-Spin potential have a expected tendency to switch the sign.

Spin-dependent potentials



Koma Koma Wittig 05, Koma Koma 06

How to calculate $c\bar{c}$ potential？

✦Heavy quark mass introduces discretization errors of **$O((ma)^n)$**

✓ At charm quark mass, it becomes severe: **m_c ~ 1.5 GeV** and **1/a ~ 2 GeV**, then **$m_c a$ ~O(1)**.

✦The Fermilab group proposed relativistic heavy quark action (RHQ) approach where all $O((ma)^n)$ errors are removed by the appropriate choice of **m_0, ξ, r_s, C_B, C_E** . **A. X. El-Khadra, A. S. Kronfeld and P. B. Mackenzie, (1997)**

$$S_{\text{lat}} = \sum_{n,n'} \bar{\psi}_{n'} (\gamma^0 D^0 + \zeta \vec{\gamma} \cdot \vec{D} + m_0 a - \frac{r_t}{2} a (D^0)^2 - \frac{r_s}{2} a (\vec{D})^2 + \sum_{i,j} \frac{i}{4} c_B a \sigma_{ij} F_{ij} + \sum_i \frac{i}{2} c_E a \sigma_{0i} F_{0i})_{n',n} \psi_n$$

We take the Tsukuba procedure in our study.

S. Aoki, Y. Kuramashi, and S.-i. Tominaga, Prog. Theor. Phys. 109, 383 (2003)

Y. Kayaba et al. [CP-PACS Collaboration], JHEP 0702, 019 (2007).

Tuning RHQ parameters

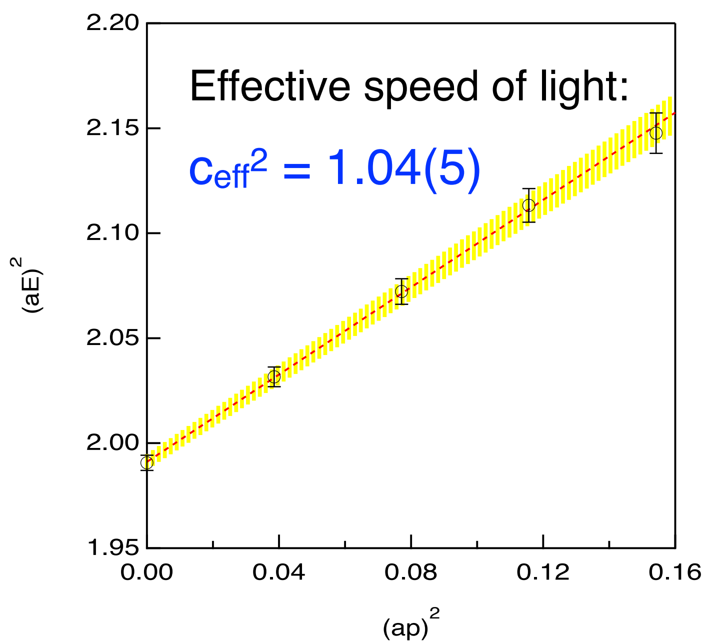
Y. Namekawa et al. [CP-PACS Collaboration], arXiv:1104.4600

RHQ action (Tsukuba-type) has 5 parameters $\varkappa_c$, ν, r_s, c_B, c_E

- The parameters r_S, c_B and c_E are determined by one-loop perturbation.
- For ν, we use a non-perturbatively determined value.

Dispersion relation: $$E^2(\mathbf{p}^2) = M^2 + c_{\rm eff}^2|\mathbf{p}|^2$$

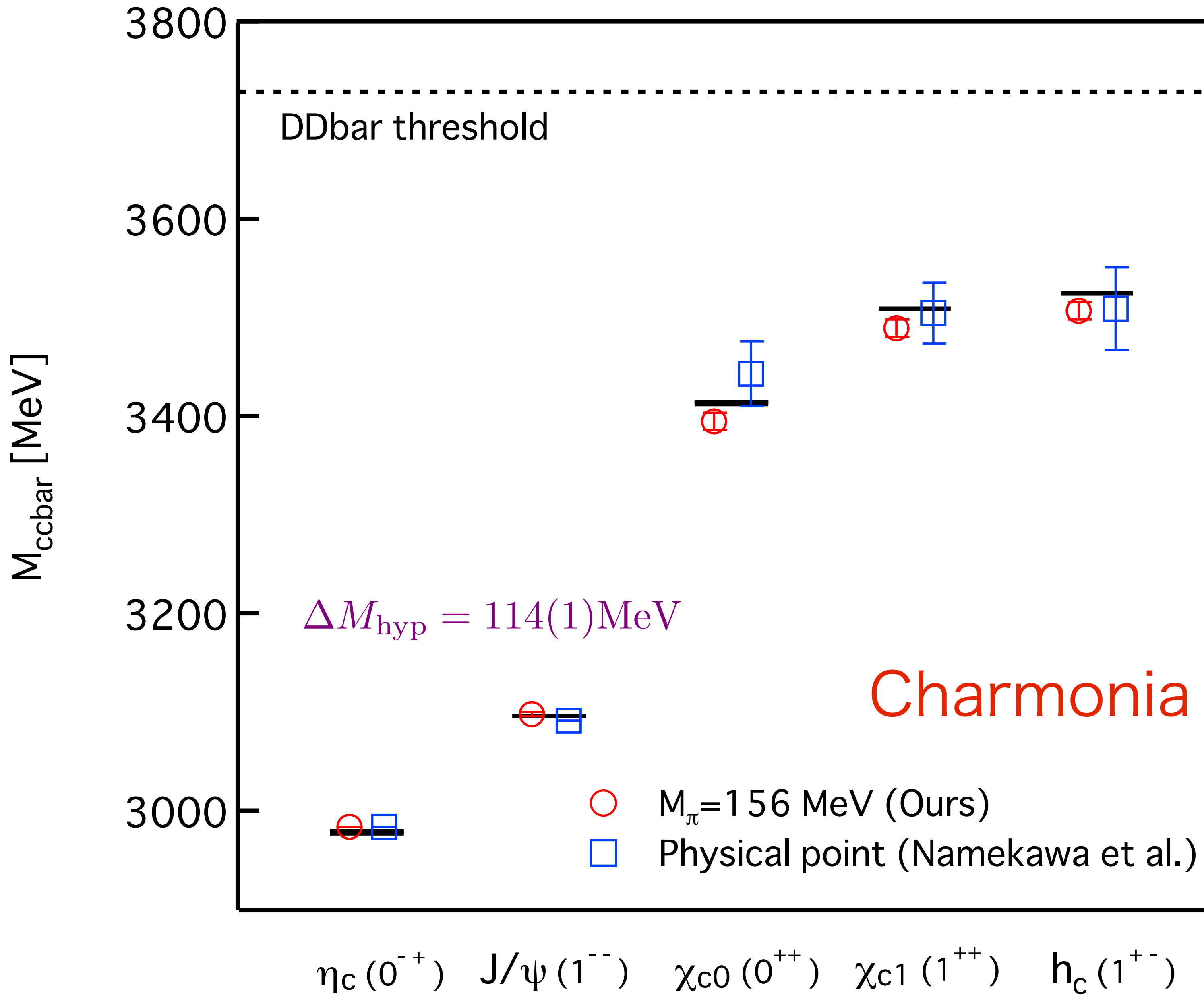
- $\varkappa_c$ is chosen to reproduce the experimental spin-averaged mass of 1S charmonium states M_{exp} = 3.0678(3) GeV.



$\varkappa_c$	ν	r_s	c_B	c_E
0.10819	1.2153	1.2131	2.0268	1.7911

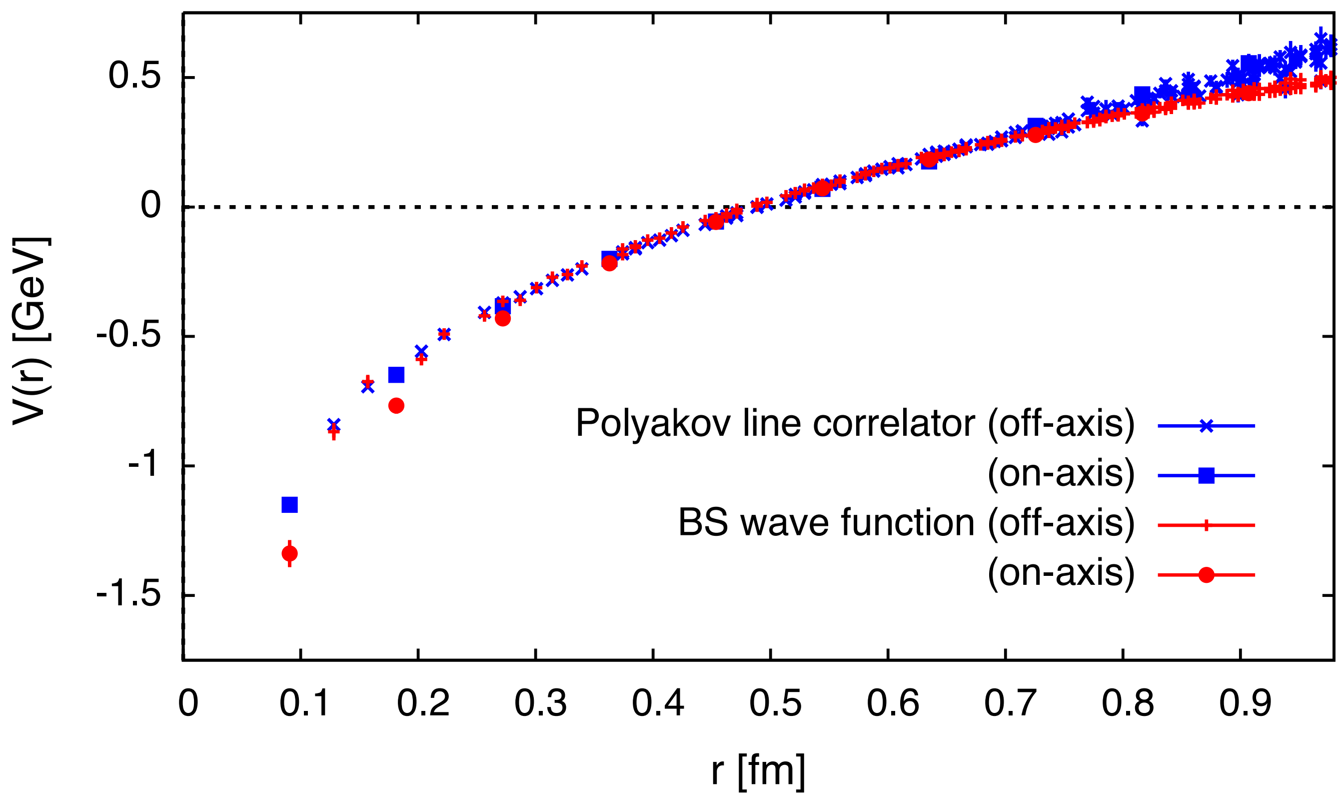
m_{ave} = 3.069(2) GeV,
m_{hyp} = 0.1110(17) GeV

cf. $m_{hyp}(exp)$ = 0.1165(12) GeV



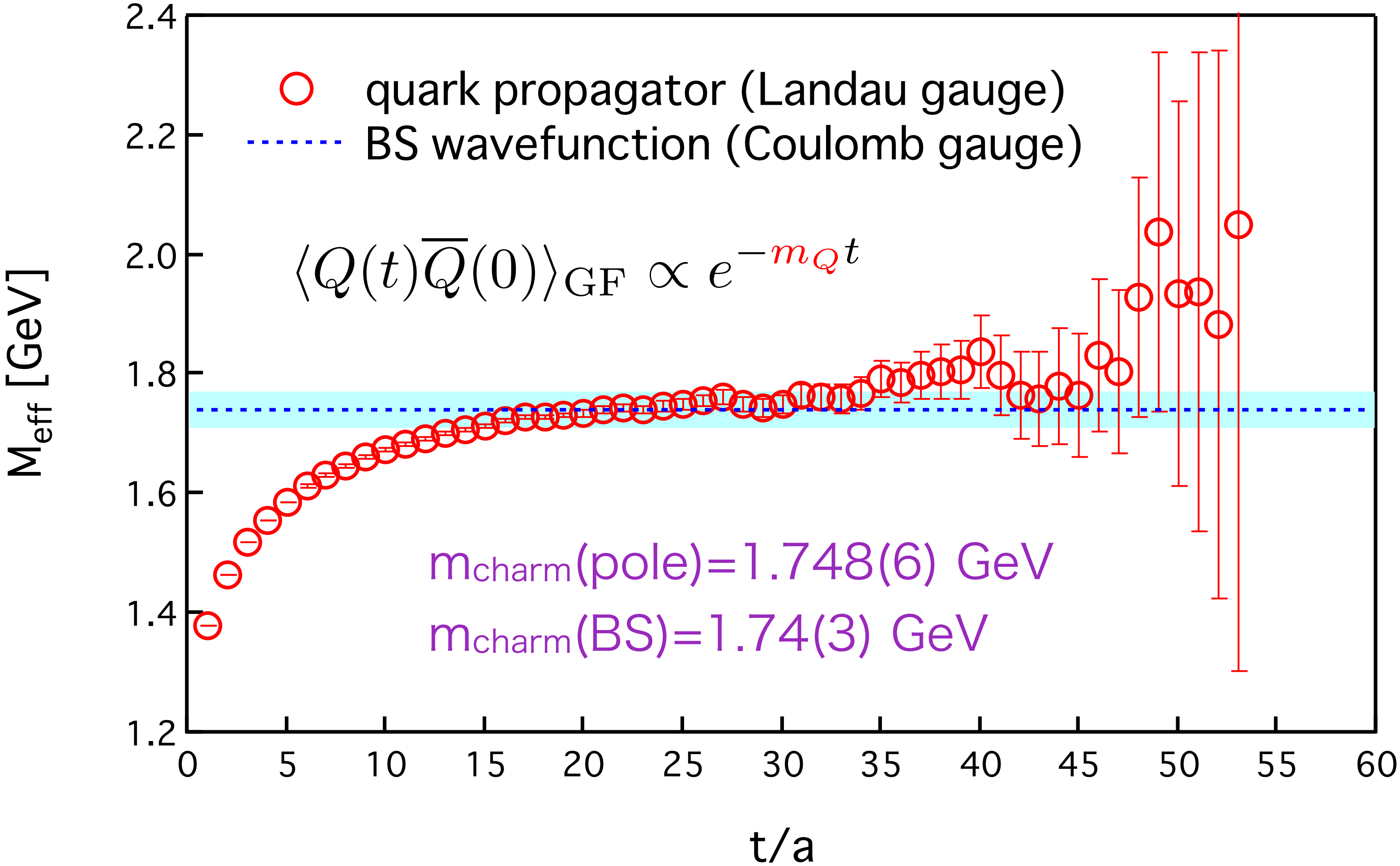
3800
3600
3400
3200
3000
DDbar threshold
M_{ccbar} [MeV]
$\Delta M_{\rm hyp} = 114(1){\rm MeV}$
Charmonia
M_π=156 MeV (Ours)
Physical point (Namekawa et al.)
η_c (0^{-+})
J/ψ (1^{--})
χ_{c0} (0^{++})
χ_{c1} (1^{++})
h_c (1^{+-})

Result; spin-independent $c\bar{c}$ potential

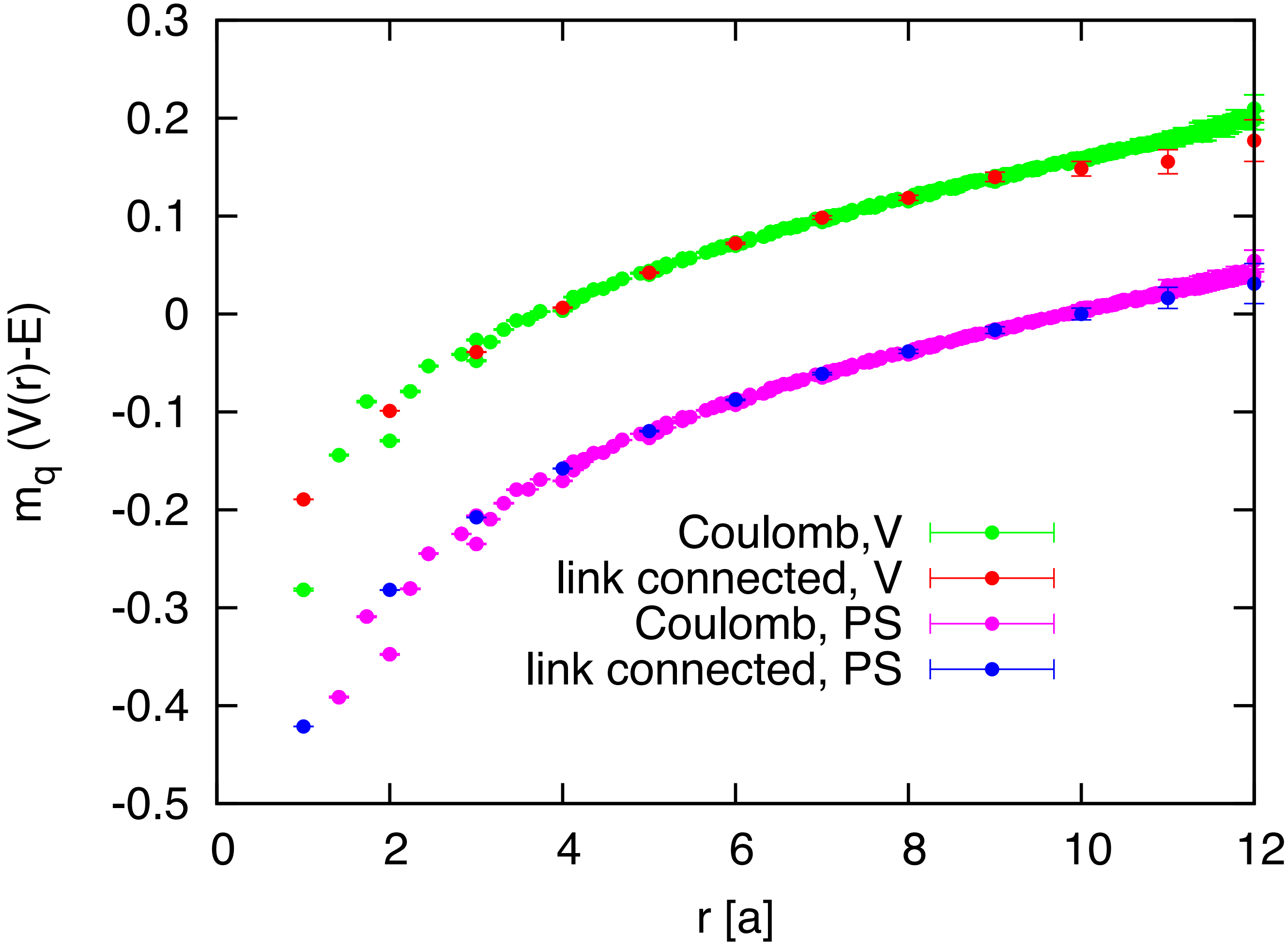


- We take a weighted average of data points in the wide range of $(t-t_{src})/a$ = 34-44
- A discretization error appears especially near the origin.

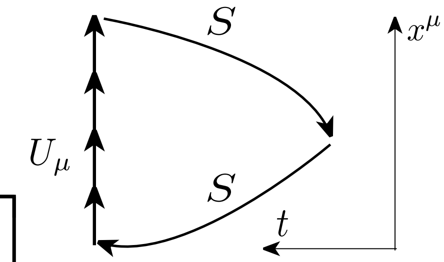
What does “quark mass” correspond to ?



Spatial information = Temporal information



0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
2
4
6
8
10
12
m_q (V(r)-E)
r [a]
Coulomb,V
link connected, V
Coulomb, PS
link connected, PS



S
S
U_μ
t
x^μ